Calculation of gluon and four-quark condensates from the operator product expansion

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The magnitudes of gluon and four-quark condensates are found from the analysis of vector mesons consisting of light quarks (the families of ρ and ω mesons) in the three-loops approximation. The QCD model with an infinite number of vector mesons is used to describe the function $R(s)$. This model describes well the experimental function $R(s)$. Polarization operators calculated with this model coincide with the Wilson operator expansion at large Q^2. The improved perturbative theory, such that the polarization operators have correct analytical properties, is used. The result is $\langle 0|(\alpha_s/\pi)G^2|0\rangle = 0.062 \pm 0.019$ GeV^4. The electronic widths of $\rho(1450)$ and $\omega(1420)$ are calculated.

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I. INTRODUCTION

The purpose of this work is to propose a new method of calculation of gluon and other condensates in some loop approximation from analysis of families of vector mesons consisting of light quarks (ρ, ω-families). Calculation of gluon and other condensates is based on the Wilson operator expansion (OPE) (Eqs. (7)–(10)) for polarization operator $[\Pi^{(\rho)}(Q^2)]_{\rm theor}$). On the other hand, the dispersion relation for $[\Pi^{(\rho)}(Q^2)]_{\rm exp}$ (Eq. (3)) makes it possible to express $[\Pi^{(\rho)}(Q^2)]_{\rm exp}$ through the measurable function $R^{(\rho)}(s)$. At large Q^2 $[\Pi^{(\rho)}(Q^2)]_{\rm exp}$ must coincide with $[\Pi^{(\rho)}(Q^2)]_{\rm theor}$. In order to present $[\Pi^{(\rho)}(Q^2)]_{\rm exp}$ in Eq. (7) we use the QCD model with an infinite number of vector mesons (MINVM) suggested and used in papers [1–3]. The MINVM was used in papers [4,5] for calculation of hadronic contribution to the muon $(g-2)$-factor and $\alpha(M_Z^2)$. The accuracy of these calculations by the MINVM is about 1%. It is evident that the MINVM can be used only under the integral.

This work is the continuation of Ref. [6] devoted to evaluating of the QCD parameters. In Ref. [6] the analyticity of QCD polarization operators was combined with the renormalization group and was used for investigation of hadronic τ-decay.

The calculation according to the renormalization group leads to the appearance of nonphysical singularities. So, the one-loop calculation gives a nonphysical pole, while in the calculation in a larger number of loops the pole disappears, but a nonphysical cut appears, $[-\Lambda_3^2, 0]$.[1] In Ref. [6] it was shown that there are only two values of Λ_3, such that theoretical predictions of QCD for $R_{\tau,V+A}$ (Eqs. (23) and (24) of [6]) agree with the experiments [7–9]. These values, calculating in the three loops, are the following: one conventional value $\Lambda_3^{\rm conv} = (618 \pm 29)$ MeV, and the other value of Λ_3 is $\Lambda_3^{\rm new} = (1666 \pm 7)$ MeV. The predictions of QCD consistent with the experiments [7–9] are just within these values of Λ_3. If one simply takes off the nonphysical cut and leaves the conventional value $\Lambda_3^{\rm conv}$ then discrepancy between the theory and the experiment will arise. As was shown in [6], if instead of the conventional value $\Lambda_3^{\rm conv}$ one chooses the value $\Lambda_3^{\rm new} = (1565 \pm 193)$ MeV, then only the physical cut contribution is sufficient to explain the experiment on hadronic τ-decay.

The new sum rules following only from analytical properties of the polarization operator were obtained in [6]. These sum rules imply that there is an essential discrepancy between perturbation theory in QCD and the experiment in hadronic τ-decay at the conventional value of Λ_3. If $\Lambda_3 = \Lambda_3^{\rm new}$, this discrepancy is absent. Because $\Lambda_3^{\rm conv}$ is conventional, we will calculate the condensates for both admissible values of Λ_3: $\Lambda_3^{\rm new} = (1565 \pm 193)$ MeV (in three loops) without the nonphysical cut corresponding $\alpha_s(-m_r^2) = 0.379 \pm 0.013$ and $\Lambda_3^{\rm conv} = (618 \pm 29)$ MeV with the nonphysical cut corresponding $\alpha_s(-m_r^2) = 0.354 \pm 0.010$.

The paper is organized as follows. In Sec. II the QCD model with an infinite number is expanded. The polarization operator defined from the experiment with the help of Eqs. (3)–(6) has the form of OPE and, owing to this, it is needless to use the Borel transformation. To find the condensates it is sufficient to equate the coefficients at $1/Q^n$. In Sec. III the magnitudes of gluon and four-quark condensates are calculated from analysis of the ρ-meson family without taking into account $\rho - \omega$ interference. It is obtained $\langle 0|(\alpha_s/\pi)G^2|0\rangle = (0.074 \pm 0.023)$ GeV^4 ($\Lambda_3 = 1.565$ GeV) and $\langle 0|(\alpha_s)/\pi G^2|0\rangle = (0.112 \pm 0.021)$ GeV^4 ($\Lambda_3 = 0.618$ GeV).

In Sec. IV the magnitudes of gluon and four-quark condensates are calculated from analysis of the ω-meson family without taking into account $\rho - \omega$ interference. It is obtained $\langle 0|(\alpha_s/\pi)G^2|0\rangle = (-0.076 \pm 0.033)$ GeV^4 ($\Lambda_3 = 1.565$ GeV) and $\langle 0|(\alpha_s/\pi)G^2|0\rangle = (-0.043 \pm 0.031)$ GeV^4 ($\Lambda_3 = 0.618$ GeV^4).

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[1]This is the definition of Λ_3.

The magnitude of gluon condensate (GC) obtained from analysis of the ω-family must be equal to the magnitude of gluon condensate from analysis of the ρ-family.

This discrepancy is due to the fact that all formulas obtained from OPE are valid only for states with pure isospin. But, owing to the vicinity of the mass of the ρ and ω-mesons the ρ-meson has a small admixture of the state with isospin $I = 0$ and the ω-meson has a small admixture of the state with isospin $I = 1$. The contradiction resolves after the separation from the ρ and ω mesons of the pure states ρ_0 with the isospin $I = 1$ and ω_0 with isospin $I = 0$. It must be emphasized that strong cancellations occur in the formulas determining the condensates. For these reasons the account of the fine effects of the $\rho - \omega$ interference is essential. In Sec. V the analysis of the ω-family with account of the $\rho - \omega$ interference is given. The results of the calculations in the 0–3 loop approximation of gluon and four-quark condensates are presented in Table I ($\Lambda_3 = 1.565$ GeV) and in Table II ($\Lambda_3 = 0.618$ GeV). The same analysis of the ρ-family is given in Sec. VI (Table III and IV). Because the values of the condensates obtained from the analysis of the ρ and ω families agree closely in Sec. IX, the averaged values of the condensates (Table V and VI) are presented. From these tables it is evident that the expansion of the gluon and 4-quark condensates in terms of α_s is very good. As a byproduct, in Sec. VII and VIII, we calculate the electronic widths of $\rho(1450)$ and $\omega(1420)$.

II. THE QCD MODEL WITH AN INFINITE NUMBER OF VECTOR MESONS

In this Section we present the QCD model with an infinite number of vector mesons suggested in Refs. [1–3]. This model is a basis for the following calculations.

Let us consider at first the family of ρ mesons. The polarization operator $\Pi^{(\rho)}$ corresponding to the ρ-meson family has the form

$$i\int d^4x e^{iqx}\langle 0|T\{j_\mu^{I=1}(x), j_\nu^{I=1}(0)\}|0\rangle = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi^{(\rho)}(Q^2), \tag{1}$$

where $Q^2 = -q^2$ and

$$j_\mu^{I=1}(x) = (1/2)[\bar{u}(x)\gamma_\mu u(x) - \bar{d}(x)\gamma_\mu d(x)] \tag{2}$$

is the current of vector mesons with isospin $I = 1$.

The dispersion relation is given by

$$\Pi^{(\rho)}(Q^2) = \frac{1}{12\pi^2}\int_{4m_\pi^2}^\infty \frac{R^{I=1}(s)}{s + Q^2} ds. \tag{3}$$

For the sake of simplicity it is written without subtractions. It is shown below that the divergent terms in (3) cancel. In the QCD model with an infinite number of narrow resonances with the masses M_k and the electronic widths Γ_k^{ee}, function $R^{I=1}(s)$ has the form

$$R^{I=1}(s) = \frac{9\pi}{\alpha^2}\sum_{k=0}^\infty \Gamma_k^{ee} M_k \delta(s - M_k^2) \tag{4}$$

where $\alpha = 1/137$. Equation (4) obviously contradicts to experiments because it assumes resonances of zero width. Let us replace $\delta(s - M_k^2) \to (1/\pi)M_k\Gamma_k/[(s - M_k^2)^2 + M_k^2\Gamma_k^2]$, where Γ_k is the total width of k-th resonance. Then we get, instead of Eq. (4),

$$R^{I=1}(s) = \frac{9}{\alpha^2}\sum_{k=0}^\infty \frac{\Gamma_k^2 M_k^2}{(s - M_k^2)^2 + M_k^2\Gamma_k^2} \tag{4a}$$

If the total widths of all resonances Γ_k are much smaller than their masses M_k, the results of the integration of (4) and (4a) with smooth functions coincide. If $M_k\Gamma_k \gg M_k^2 - M_{k-1}^2$ for $k > 3$ Eq. (4a) is described by a smooth curve for $s > M_3^2$. When fulfilling these conditions Eq. (4a) is consistent with experimental data of $R^{I=1}(s)$. Equation (4) will be used only under the integral with a smooth function.

Using (4), (3) is recast into the form

$$\Pi^{(\rho)}(Q^2) = \frac{3}{4\pi\alpha^2}\sum_{k=0}^\infty \frac{\Gamma_k^{ee} M_k}{s_k + Q^2}. \tag{5}$$

The polarization operator (5) can be rearranged into a form with the separated unit operator. The remainder of the polarization operator can be associated with gluon condensate and with contribution of higher dimensional operators. To do this we transform the sum in (5) into an integral by means of the Euler-Maclaurin formula [10] beginning from $k = 1$. We have

$$\begin{aligned}\Pi^{(\rho)}(Q^2) = \frac{3}{4\pi\alpha^2}\Bigg\{&\int_{(m_u+m_d)^2}^\infty \frac{\Gamma_k^{ee}M_k}{s_k + Q^2}\frac{dk}{ds_k}ds_k \\ &- \int_{(m_u+m_d)^2}^{s_1} \frac{\Gamma_k^{ee}M_k}{s_k + Q^2}\frac{dk}{ds_k}ds_k + \frac{\Gamma_0^{ee}M_0}{s_0 + Q^2} \\ &+ \frac{1}{2}\frac{\Gamma_1^{ee}M_1}{s_1 + Q^2} - \frac{1}{12}\frac{d}{dk}\left(\frac{\Gamma_k^{ee}M_k}{s_k + Q^2}\right)\Bigg|_{k=1} \\ &+ \frac{1}{720}\frac{d^3}{dk^3}\left(\frac{\Gamma_k^{ee}M_k}{s_k + Q^2}\right)\Bigg|_{k=1} - \cdots\Bigg\}.\end{aligned} \tag{6}$$

The operator expansion for $\Pi^{(\rho)}$, which is valid at high Q^2 has the form [11]

$$\Pi^{(\rho)}(Q^2) = \int_{(m_u+m_d)^2}^\infty \frac{R_{PT}^{I=1}(s)ds}{s + Q^2} + C_2/Q^2 + C_4/Q^4 + C_6/Q^6 + \cdots \tag{7}$$

$$C_2 = 0 \tag{8}$$

$$C_4 = \frac{1}{24}\left\langle 0 \left| \frac{\alpha_s}{\pi} G^2 \right| 0 \right\rangle + \frac{1}{2}(m_u\langle 0|\bar{u}u|0\rangle + m_d\langle 0|\bar{d}d|0\rangle) \tag{9}$$

$$C_6 = -\frac{1}{2}\pi\alpha_s\langle 0|(\bar{u}\gamma_\mu\gamma_5 t^a u - \bar{d}\gamma_\mu\gamma_5 t^a d)^2|0\rangle - \frac{1}{9}\pi\alpha_s\left\langle 0 \left| (\bar{u}\gamma_\mu t^a u + \bar{d}\gamma_\mu t^a d) \times \sum_{q=u,d,s} \bar{q}\,\gamma_\mu t^a q \right| 0 \right\rangle. \tag{10}$$

In the region where the operator expansion is valid, $\Pi^{(\rho)}(Q^2)$ should not differ significantly from $\Pi^{(\rho)}_{\rm theor}(Q^2)$ (see Eq. (7)). For this reason we equate the first term on the right-hand part of (6) to the first term on the right-hand part of (7):

$$\frac{3}{4\pi\alpha^2}\int_{(m_u+m_d)^2}^{\infty} \frac{\Gamma_k^{ee} M_k}{s_k + Q^2}\frac{dk}{ds_k} ds_k = \frac{1}{12\pi^2}\int_{(m_u+m_d)^2}^{\infty} \frac{R_{PT}^{I=1}(s)ds}{s+Q^2}. \tag{11}$$

We consider the equality (11) as an ansatz that makes it possible to separate large terms associated with the unit operator from small terms associated with condensates and consider it as an equation for Γ_k^{ee}. Equation (11) has one and only one solution[2]:

$$\Gamma_k^{ee} = \frac{2\alpha^2}{9\pi} R_{PT}^{(I=1)}(s_k) M_k^{(1)}. \tag{12}$$

This result follows from uniqueness of the theorem for analytic functions. The solution (12) is obtained by equating the jumps on the cut in (11). In Eq. (12) and in the following formulas we use the notation

$$s_k = M_k^2, \qquad M_k^{(l)} \equiv d^l M_k/dk^l, \qquad s_k^{(l)} \equiv d^l s_k/dk^l. \tag{13}$$

It can be proved that the function $s_k \equiv s(k)$ specified at the points $k = 0, 1, 2, \ldots$ can be extended to an analytic function of the complex variable k with a cut along the negative axis [3]. We will not employ the analytic properties of the function $s(k)$. It is assumed that the function $s(k)$ is continuous, differentiable with respect to k at $k = 1$. The derivatives $s_1^{(l)}$ will be considered as parameters.

The QCD model with an infinite number of vector mesons was used in [5,12] to calculate the contribution of strong interaction to anomalous magnetic moment of muon. The result was found to be

$$a_\mu^{\rm hadr} = [(g-2)/2]_{\rm hadr} = \frac{\alpha}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds K(s) R(s)/s = 678(7)\times 10^{-10}. \tag{14}$$

The result given by Eq. (14) should be compared with the recent, more precise, $a_\mu^{\rm hadr}$ value calculated by integrating Eq. (14) with the cross sections measured from annihilation of e^+e^- to hadrons [13–15]

$$\begin{aligned} a_\mu^{\rm hadr} &= 6847(70)\times 10^{-11} \quad [13] \\ a_\mu^{\rm hadr} &= 6831(61)\times 10^{-11} \quad [14] \\ a_\mu^{\rm hadr} &= 6909(64)\times 10^{-11} \quad [15]. \end{aligned} \tag{15}$$

In addition, the QCD model with an infinite number of vector mesons was used in [4,5,12] to calculate the contribution of strong interaction to quantity $\alpha(M_z^2)$. It was found that

$$\delta\alpha_{\rm hadr} = \frac{\alpha M_z^2}{3\pi} P \int_{4m_\pi^2}^{\infty} \frac{R(s)ds}{(M_z^2 - s)s} = 0.02786(6). \tag{16}$$

This result should be compared with the results $\delta\alpha_{\rm hadr} = 0.02744(36)$ [16], 0.02803(65) [17], 0.0280(7) [18], 0.02754(46) [19], 0.02737(39) [20], 0.02784(22) [21], 0.02778(16) [22], 0.02779(20) [23], 0.02770(15) [24], 0.02787 (32) [25], 0.02778(24) [26], 0.02741 (19) [27], 0.02763(36) [28], and 0.02747(12) [15], which were obtained by calculating the integral (16) with the experimental cross section from e^+e^- into hadrons. We emphasize that the quantity $\delta\alpha_{\rm hadr}$ is calculated in (16) with the highest accuracy.

It is important to note that the integrals which describe hadronic contributions to the $(g-2)$ factor for muon and to $\alpha(M_z^2)$ are determined by different regions: the integrals for hadronic contributions to the $(g-2)$ factor are governed by the region of small $s \sim m_\rho^2$, while the integral for hadronic contribution to $\alpha(M_z^2)$ is dominated by large $s \sim M_z^2$. From the above, the accuracy of calculations by MINVM is about 1%. This accuracy is sufficient for calculations of gluon and four-quark condensates. It is obvious that MINVM can be used only under the integral.

III. MAGNITUDE OF GLUON AND FOUR-QUARK CONDENSATES FROM ANALYSIS OF ρ-MESON FAMILY

At present, three mesons of the ρ family with the masses $M_0 = 0.7711 \pm 0.0009$ GeV, $M_1 = 1.465 \pm 0.025$ GeV, $M_2 = 1.7 \pm 0.02$ GeV have been found. The electronic width of $\rho(770)$ is $\Gamma_0^{ee} = (6.85 \pm 0.11)$ keV [30].

Comparing (6) and (7) at high Q^2 and using Eq. (12), we arrive at

[2]The analogous formula in nonrelativistic quantum mechanics

$$|\psi_k(0)|^2 = \frac{m^{3/2}}{\sqrt{2}\pi^2} E_k^{1/2}\frac{dE_k}{dk} \tag{12a}$$

has accuracy $\lesssim 2\%$ for usual potential.

$$C_2 = \frac{1}{12\pi^2}\left\{-\int_{(m_u+m_d)^2}^{s_1} R_{PT}^{I=1}(s)ds + R_{PT}^{I=1}(s_0)s_0^{(1)} + \frac{1}{2}R_{PT}^{I=1}(s_1)s_1^{(1)} - \frac{1}{12}[R_{PT}^{I=1}(s_1)s_1^{(1)}]^{(1)}\right\} \quad (17)$$

$$C_4 = \frac{1}{12\pi^2}\left\{\int_{(m_u+m_d)^2}^{s_1} R_{PT}^{I=1}(s)sds - R_{PT}^{I=1}(s_0)s_0s_0^{(1)} - \frac{1}{2}R_{PT}^{I=1}(s_1)s_1s_1^{(1)} + \frac{1}{12}[R_{PT}^{I=1}(s_1)s_1s_1^{(1)}]^{(1)}\right\} \quad (18)$$

$$C_6 = \frac{1}{12\pi^2}\left\{-\int_{(m_u+m_d)^2}^{s_1} R_{PT}^{I=1}(s)s^2ds + R_{PT}^{I=1}(s_0)s_0^2s_0^{(1)} + \frac{1}{2}R_{PT}^{I=1}(s_1)s_1^2s_1^{(1)} - \frac{1}{12}[R_{PT}^{I=1}(s_1)s_1^2s_1^{(1)}]^{(1)}\right\}. \quad (19)$$

We discard the terms with small coefficients 1/720 and 1/30240 in Eqs. (17)–(19). Let us write $R_{PT}^{I=1}$ in the form

$$R_{PT}^{I=1}(s) = \frac{3}{2}[1 + r(s)]. \quad (20)$$

Function $r(s)$ is calculated in [6]. Neglecting the terms associated with u and d-quark masses we get

$$C_2 = \frac{1}{8\pi^2}\left\{-s_1 + s_0^{(1)} + \frac{1}{2}s_1^{(1)} - \frac{1}{12}s_1^{(2)} - \int_0^{s_1} r(s)ds + r(s_0)s_0^{(1)} + \frac{1}{2}r(s_1)s_1^{(1)} - \frac{1}{12}[r(s_1)s_1^{(1)}]^{(1)}\right\} = 0 \quad (21)$$

$$C_4 = \frac{1}{8\pi^2}\left\{\frac{1}{2}s_1^2 - s_0s_0^{(1)} - \frac{1}{2}s_1s_1^{(1)} + \frac{1}{12}(s_1s_1^{(1)})^{(1)} + \int_0^{s_1} sr(s)ds - s_0s_0^{(1)}r(s_0) - \frac{1}{2}s_1s_1^{(1)}r(s_1) + \frac{1}{12}[s_1s_1^{(1)}r(s_1)]^{(1)}\right\} \quad (22)$$

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$$C_6 = \frac{1}{8\pi^2}\left\{-\frac{1}{3}s_1^3 + s_0^2s_0^{(1)} + \frac{1}{2}s_1^2s_1^{(1)} - \frac{1}{12}(s_1^2s_1^{(1)})^{(1)} - \int_0^{s_1} s^2r(s)ds + s_0^2s_0^{(1)}r(s_0) + \frac{1}{2}s_1^2s_1^{(1)}r(s_1) - \frac{1}{12}[s_1^2s_1^{(1)}r(s_1)]^{(1)}\right\}. \quad (23)$$

Using (21) to eliminate the unobservable quantity $s_1^{(2)}$ we reduce (22) and (23) to the form

$$\left\langle 0\left|\frac{\alpha_s}{\pi}G^2\right|0\right\rangle = \frac{3}{\pi^2}\left\{-\frac{1}{2}s_1^2 + (s_1 - s_0)s_0^{(1)} + \frac{1}{12}s_1^{(1)^2} - \int_0^{s_1}(s_1 - s)r(s)ds + (s_1 - s_0)s_0^{(1)}r(s_0) + \frac{1}{12}s_1^{(1)^2}r(s_1)\right\} \quad (24)$$

$$C_6 = \frac{1}{8\pi^2}\left\{\frac{2}{3}s_1^3 - (s_1^2 - s_0^2)s_0^{(1)} - \frac{1}{6}s_1s_1^{(1)^2} + \int_0^{s_1}(s_1^2 - s^2)r(s)ds - (s_1^2 - s_0^2)s_0^{(1)}r(s_0) - \frac{1}{6}s_1s_1^{(1)^2}r(s_1)\right\}. \quad (25)$$

Let us calculate $s_0^{(1)}$ and $s_1^{(1)}$. We obtain from (12) and (20)

$$s_k^{(1)} = \frac{6\pi\Gamma_k^{ee}M_k}{\alpha^2[1 + r(s_k)]}. \quad (26)$$

We know only Γ_0^{ee}, therefore

$$s_0^{(1)} = (1.554 \pm 0.024)\ \text{GeV}^2 \quad [\Lambda_3 = 1.565\ \text{GeV}, \quad r(s_0) = 0.204] \quad (27a)$$

$$s_0^{(1)} = (1.634 \pm 0.026)\ \text{GeV}^2 \quad [\Lambda_3 = 0.618\ \text{GeV}, \quad r(s_0) = 0.144]. \quad (27b)$$

The quantity $s_1^{(1)}$ is determined from trivial equations

$$s_2 = s_1 + s_1^{(1)} + \frac{1}{2}s_1^{(2)} + \frac{1}{6}s_1^{(3)} \quad (28)$$

$$s_0 = s_1 - s_1^{(1)} + \frac{1}{2}s_1^{(2)} - \frac{1}{6}s_1^{(3)}. \quad (29)$$

From (28) and (29) we obtain

$$s_1^{(1)} = \frac{s_2 - s_0}{2} - \frac{1}{6}s_1^{(3)} \quad (30)$$

and

$$s_1^{(2)} = s_0 + s_2 - 2s_1 = -0.81 \pm 0.16\ \text{GeV}^2. \quad (31)$$

To estimate the last term in (30), we note that $|s_1^{(2)}|$ is smaller than $s_1^{(1)}$ (see (32)). We put $|s_1^{(3)}| = |s_1^{(2)}|$ and include the last term in (30) into the error in $s_1^{(1)}$ and obtain finally

$$s_1^{(1)} = \frac{s_2 - s_0}{2} = 1.148 \pm 0.139 \text{ GeV}^2. \tag{32}$$

Using Eqs. (24) and (25) and the values of $s_0^{(1)}$ and $s_1^{(1)}$ from (27a), (27b), and (32), we find that the analysis of the ρ-meson family leads, in the 3-loop approximation, to the following results for gluon and four-quark condensates:

$$\left\langle 0 \left| \frac{\alpha_s}{\pi} G^2 \right| 0 \right\rangle = (0.0744 \pm 0.0227) \text{ GeV}^4, \qquad (\Lambda_3 = 1.565 \text{ GeV}) \tag{33a}$$

$$\left\langle 0 \left| \frac{\alpha_s}{\pi} G^2 \right| 0 \right\rangle = (0.112 \pm 0.021) \text{ GeV}^4, \qquad (\Lambda_3 = 0.618 \text{ GeV}) \tag{33b}$$

$$C_6 = -0.0072 \pm 0.0041 \text{ GeV}^6, \qquad (\Lambda_3 = 1.565 \text{ GeV}) \tag{34a}$$

$$C_6 = -0.0115 \pm 0.0034 \text{ GeV}^6, \qquad (\Lambda_3 = 0.618 \text{ GeV}). \tag{34b}$$

The errors presented in (33a), (33b), (34a), and (34b) were found from the formulas

$$\Delta\left\langle 0 \left| \frac{\alpha_s}{\pi} G^2 \right| 0 \right\rangle = \frac{3}{\pi^2}\Bigg(\{(s_1 - s_0)[1 + r(s_0)]\Delta s_0^{(1)}\}^2 + \left\{\left[s_0^{(1)} - s_1 - \int_0^{s_1} r(s)ds + s_0^{(1)} r(s_0)\right]\Delta s_1\right\}^2 + \left\{\frac{1}{6} s_1^{(1)}[1 + r(s_1)]\Delta s_1^{(1)}\right\}^2\Bigg)^{1/2} \tag{35}$$

$$\Delta C_6 = \frac{1}{8\pi^2}\Bigg(\left\{\left[2s_1^2 - 2s_1 s_0^{(1)} - \frac{1}{6} s_1^{(1)^2} + 2s_1 \int_0^{s_1} r(s)ds\right]\Delta s_1\right\}^2 + \{(s_1^2 - s_0^2)[1 + r(s_0)]\Delta s_0^{(1)}\}^2 + \left\{\frac{1}{3} s_1 s_1^{(1)}[1 + r(s_1)]\Delta s_1^{(1)}\right\}^2\Bigg)^{1/2}. \tag{36}$$

The symbol Δ in Eqs. (35) and (36) denotes the error in the corresponding quality (for example, $\Delta s_1^{(1)} = 0.139 \text{ GeV}^2$). The small error connected with the error in Λ_3 is taken into account at numerical calculations.

IV. MAGNITUDE OF GLUON CONDENSATE FROM THE ANALYSIS OF THE ω-MESON FAMILY

The polarization operator associated with the isoscalar current $j_\mu^{I=0}(x)$ of light quarks can be written as

$$i\int d^4x e^{iqx}\langle 0|T\{j_\mu^{I=0}(x) j_\nu^{I=0}(0)\}|0\rangle = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi^{(w)}(Q^2) \tag{37}$$

where

$$j_\mu^{I=0}(x) = \frac{1}{6}\{\overline{u}(x)\gamma_\mu u(x) + \overline{d}(x)\gamma_\mu d(x)\}. \tag{38}$$

With the exception of (12), all the equations presented above for the ρ-meson family remain in force. The equation that takes place in (12) for the ω-meson family is

$$\Gamma_k^{ee} = \frac{1}{9}\frac{\alpha^2}{3\pi}[1 + r(s_k)]M_k^{(1)}. \tag{39}$$

Three ω mesons—the ω meson with the mass $M_0 = 0.78257 \pm 0.00012$ GeV and the electronic width $\Gamma_0^{ee} = 0.60 \pm 0.02$ keV, the ω' meson with the mass $M_1 = 1.419 \pm 0.031$ GeV, and ω'' meson with the mass $M_2 = 1.649 \pm 0.024$ GeV [30]—have been discovered thus far. Proceeding in the same way as for the ρ-meson family and taking into account (39), we obtain

$$s_0^{(1)} = (1.244 \pm 0.041) \text{ GeV}^2, \qquad \Lambda_3 = 1.565 \text{ GeV} \tag{40a}$$

$$s_0^{(1)} = (1.308 \pm 0.044) \text{ GeV}^2, \qquad \Lambda_3 = 0.618 \text{ GeV} \tag{40b}$$

$$s_1^{(1)} = (1.053 \pm 0.123) \text{ GeV}^2. \tag{40c}$$

Using Eqs. (24) and (35) and the values in (40a)–(40c) we obtain

$$\langle 0|(\alpha_s/\pi)G^2|0\rangle = (-0.076 \pm 0.033) \text{ GeV}^4 \qquad (\Lambda_3 = 1.565 \text{ GeV}) \tag{41a}$$

$$\langle 0|(\alpha_s/\pi)G^2|0\rangle = (-0.043 \pm 0.031) \text{ GeV}^4 \qquad (\Lambda_3 = 0.618 \text{ GeV}) \tag{41b}$$

TABLE I. The results of the calculations of the gluon and 4-quark condensates in the 0–3 loops approximation for $\Lambda_3^{\rm new}$ from analysis of the ω-meson family taking into account the $\rho - \omega$ interference.

	$\Lambda_3^{\rm new}$/GeV	$\frac{\langle 0\|(\alpha_s/\pi)G^2\|\rangle}{\rm GeV^4}$	$C_6/{\rm GeV}^6$	$r(m_\omega^2)$	$r(m_{\omega l}'^2)$
0 loops		0.198 ± 0.100	-0.0218 ± 0.0034	0	0
one loop	0.618 ± 0.059	0.070 ± 0.034	-0.0082 ± 0.0050	0.201 ± 0.008	0.153 ± 0.007
two loops	1.192 ± 0.136	0.072 ± 0.034	-0.0083 ± 0.0050	0.203 ± 0.008	0.161 ± 0.008
three loops	1.565 ± 0.193	0.073 ± 0.034	-0.0084 ± 0.0050	0.202 ± 0.008	0.164 ± 0.008

$$C_6 = (0.0079 \pm 0.0052)\ {\rm GeV}^6 \qquad (\Lambda_3 = 1.565\ {\rm GeV}) \tag{42a}$$

$$C_6 = (0.0042 \pm 0.0045)\ {\rm GeV}^6 \qquad (\Lambda_3 = 0.618\ {\rm GeV}). \tag{42b}$$

The magnitude of the gluon condensate obtained from different processes must be equal. We found that the magnitude of the gluon condensate determined from the analysis of the ω-meson family is inconsistent with the magnitude of the gluon condensate from the analysis of the ρ family. The way out of this situation is proposed in the next sections.

V. $\rho - \omega$-INTERFERENCE AND RESOLUTION OF THE CONTRADICTION

Eqs. (37)–(39) are valid for isospin-zero vector mesons. However, there is a noticeable isospin-1 admixture in the real ω meson. This is obvious, for example, from the fact that the width $[\Gamma(\omega \to 2\pi)]$ is nonzero,

$$\Gamma(\omega \to 2\pi) = 0.143 \pm 0.024\ {\rm MeV} \quad [30].$$

Moreover, the ratio of the electronic widths of ρ and ω mesons is $\Gamma_\rho^{ee}/\Gamma_\omega^{ee} = 11.42(1 \pm 0.037)$, instead of the expected value nine. Let us represent the ω meson and ρ states as

$$|\omega\rangle = |\omega_0\rangle + \lambda|\rho_0\rangle \qquad |\rho\rangle = |\rho_0\rangle - \lambda|\omega_0\rangle \tag{43}$$

where $|\omega_0\rangle$ corresponds to the $I = 0$ state and $|\rho_0\rangle$ corresponds to the $I = 1$ state.

In the narrow resonance approximation the parameter λ must be real. Because total width of ρ-meson is larger than $m_\omega - m_\rho$ we do not assume that λ is real. It follows from Eq. (43) that

$$\langle \pi^+\pi^-|\omega\rangle = \lambda\langle \pi^+\pi^-|\rho_0\rangle, \tag{44}$$

$$\langle \pi^+\pi^-|\rho\rangle = \langle \pi^+\pi^-|\rho_0\rangle. \tag{45}$$

From Eqs. (44) and (45) for the value of $|\lambda|^2$ we obtain

$$|\lambda|^2 = \left|\frac{\langle \pi^+\pi^-|\omega\rangle}{\langle \pi^+\pi^-|\rho\rangle}\right|^2 = \frac{\Gamma(\omega \to 2\pi)}{\Gamma(\rho \to 2\pi)} = 0.000962 \pm 0.000158. \tag{46}$$

The values $\Gamma(\omega \to 2\pi)$ and $\Gamma(\rho \to 2\pi)$ are taken from Particle Data [30]. It follows from the formula $\langle e^+e^-|\rho_0\rangle = 3\langle e^+e^-|\omega_0\rangle$ and the formula (43) that

$$\langle e^+e^-|\omega\rangle = (1 + 3\lambda)\langle e^+e^-|\omega_0\rangle, \tag{47}$$

$$\langle e^+e^-|\rho\rangle = (3 - \lambda)\langle e^+e^-|\omega_0\rangle. \tag{48}$$

It follows from Eqs. (46)–(48) that

$$\frac{\Gamma_\rho^{ee}}{\Gamma_\omega^{ee}} = \left|\frac{\langle e^+e^-|\rho\rangle}{\langle e^+e^-|\omega\rangle}\right|^2 = \left|\frac{3-\lambda}{1+3\lambda}\right|^2 = \frac{9 - 6Re\lambda + |\lambda|^2}{1 + 6Re\lambda + 9|\lambda|^2} = 11.417 \pm 0.422. \tag{49}$$

From Eqs. (46) and (49) we obtain

$$Re\lambda = -0.0337 \pm 0.0022. \tag{50}$$

The mixing parameter λ obtained from Eqs. (46) and (49) is practically real in contrast with Feynman's book [31]. The electronic width of the pure state $|\omega_0\rangle$ is

$$\Gamma_{\omega_0}^{ee} = \frac{\Gamma_\omega^{ee}}{1 + 6Re\lambda} = (0.758 \pm 0.0028)\ {\rm KeV}. \tag{51}$$

Instead of the values $s_0^{(1)}$ from Eqs. (40a) and (40b) we get

TABLE II. The results of the calculations of the gluon and 4-quark condensates in the 0–3 loops approximation for $\Lambda_3^{\rm conv}$ from the analysis of the ω-meson family taking into account the $\rho - \omega$ interference.

	$\Lambda_3^{\rm conv}$/GeV	$\frac{\langle 0\|(\alpha_s/\pi)G^2\|\rangle}{\rm GeV^4}$	$C_6/{\rm GeV}^6$	$r(m_\omega^2)$	$r(m_{\omega l}'^2)$
0 loops		0.198 ± 0.100	-0.0218 ± 0.0034	0	0
one loop	0.370 ± 0.019	0.097 ± 0.033	-0.0111 ± 0.0046	0.159 ± 0.004	0.122 ± 0.003
two loops	0.539 ± 0.025	0.105 ± 0.033	-0.0119 ± 0.0045	0.148 ± 0.003	0.114 ± 0.002
three loops	0.618 ± 0.29	0.108 ± 0.033	-0.0123 ± 0.0044	0.143 ± 0.003	0.111 ± 0.002

TABLE III. The results of the calculations of the gluon and 4-quark condensates in the 0–3 loops approximation for Λ_3^{new} from analysis of the ρ-meson family taking into account the $\rho - \omega$ interference.

	$\Lambda_3^{\rm new}$/GeV	$\frac{\langle 0\vert(\alpha_s/\pi)G^2\vert\rangle}{\rm GeV^4}$	C_6/GeV^6	$r(m_\rho^2)$	$r(m_{\rho l}^{\prime 2})$
0 loops		0.197 ± 0.059	-0.0211 ± 0.0025	0	0
one loop	0.618 ± 0.059	0.049 ± 0.023	-0.0044 ± 0.0045	0.202 ± 0.008	0.151 ± 0.007
two loops	1.192 ± 0.136	0.055 ± 0.023	-0.0050 ± 0.0045	0.204 ± 0.008	0.159 ± 0.008
three loopps	1.565 ± 0.193	0.056 ± 0.023	-0.0051 ± 0.0045	0.203 ± 0.008	0.162 ± 0.008

the new values

$$s_0^{(1)} = (1.535 \pm 0.057)\ \mathrm{GeV}^2, \qquad (\Lambda_3 = 1.565\ \mathrm{GeV}) \tag{52a}$$

$$s_0^{(1)} = (1.596 \pm 0.026)\ \mathrm{GeV}^2, \qquad (\Lambda_3 = 0.618\ \mathrm{GeV}) \tag{52b}$$

corresponding to the contribution of the isospin $I = 0$ in the ωmeson.

The magnitude of gluon condensate following from the analysis of the ω family is

$$\left\langle 0 \left| \frac{\alpha_s}{\pi} G^2 \right| 0 \right\rangle = (0.073 \pm 0.034)\ \mathrm{GeV}^4, \quad (\Lambda_3 = 1.565\ \mathrm{GeV}) \tag{53a}$$

$$\left\langle 0 \left| \frac{\alpha_s}{\pi} G^2 \right| 0 \right\rangle = (0.108 \pm 0.033)\ \mathrm{GeV}^4, \quad (\Lambda_3 = 0.618\ \mathrm{GeV}). \tag{53b}$$

This GC magnitude is consistent with the value which follows from the analysis of the ρ-meson family.

For C_6 we have

$$C_6 = (-0.0084 \pm 0.0050)\ \mathrm{GeV}^6, \qquad (\Lambda_3 = 1.565\ \mathrm{GeV}) \tag{54}$$

$$C_6 = (-0.0123 \pm 0.0044)\ \mathrm{GeV}^6, \qquad (\Lambda_3 = 0.618\ \mathrm{GeV}). \tag{55}$$

The results of the calculations of gluon and 4-quark condensates in 0–3 loops approximation from the analysis of the ω-meson family are presented in Tables I and II.

VI. THE MAGNITUDE OF GLUON AND FOUR-QUARK CONDENSATES FROM ρ-MESON FAMILY WITH THE ACCOUNT OF $\rho - \omega$-INTERFERENCE

Because of $\lambda \neq 0$ there is a small admixture of isospin $I = 0$ into the $\rho(770)$ meson. The electronic width of the $\rho(770)$ meson due to isospin $I = 1$ is equal to

$$\Gamma_{\rho_0}^{ee} = \frac{\Gamma_\rho^{ee}}{(1 - \frac{\lambda}{3})^2} = 6.71 \pm 0.11\ \mathrm{keV}.$$

The corrected value $s_0^{(1)}$ is equal to

$$s_0^{(1)} = (1.521 \pm 0.025)\ \mathrm{GeV}^2, \qquad (\Lambda_3 = 1.565\ \mathrm{GeV}) \tag{57a}$$

$$s_0^{(1)} = (1.596 \pm 0.026)\ \mathrm{GeV}^2, \qquad (\Lambda_3 = 0.618\ \mathrm{GeV}). \tag{57b}$$

The corresponding GC magnitude is

$$\left\langle 0 \left| \frac{\alpha_s}{\pi} G^2 \right| 0 \right\rangle = (0.056 \pm 0.023)\ \mathrm{GeV}^4, \quad (\Lambda_3 = 0.565\ \mathrm{GeV}) \tag{58a}$$

$$\left\langle 0 \left| \frac{\alpha_s}{\pi} G^2 \right| 0 \right\rangle = (0.096 \pm 0.021)\ \mathrm{GeV}^4, \quad (\Lambda_3 = 0.618\ \mathrm{GeV}). \tag{58b}$$

For C_6 we have

$$C_6 = (-0.0051 \pm 0.0045)\ \mathrm{GeV}^6, \qquad (\Lambda_3 = 1.565\ \mathrm{GeV}) \tag{59a}$$

$$C_6 = (-0.0097 \pm 0.0039)\ \mathrm{GeV}^6, \qquad (\Lambda_3 = 0.618\ \mathrm{GeV}). \tag{59b}$$

The results of the calculations of gluon and 4-quark

TABLE IV. The results of the calculations of the gluon and 4-quark condensates in the 0–3 loops approximation for Λ_3^{conv} from analysis of the ρ-meson family taking into account the $\rho - \omega$ interference.

	$\Lambda_3^{\rm new}$/GeV	$\frac{\langle 0\vert(\alpha_s/\pi)G^2\vert\rangle}{\rm GeV^4}$	C_6/GeV^6	$r(m_\rho^2)$	$r(m_{\rho l}^{\prime 2})$
0 loops		0.197 ± 0.059	-0.0211 ± 0.0025	0	0
one loop	0.370 ± 0.019	0.070 ± 0.034	-0.0082 ± 0.0050	0.159 ± 0.006	0.151 ± 0.007
two loops	1.539 ± 0.025	0.093 ± 0.022	-0.0094 ± 0.0039	0.149 ± 0.003	0.113 ± 0.002
three loops	1.618 ± 0.029	0.096 ± 0.021	-0.097 ± 0.00039	0.144 ± 0.003	0.109 ± 0.002

TABLE V. The averaged results of the calculation of the gluon and 4-quark condensates in 0–3 loops approximation for $\Lambda_3^{\rm new}$ from analysis of the ρ and ω families taking into account the $\rho - \omega$ interference.

	$\Lambda_3^{\rm new}$/GeV	$\frac{\langle 0|(\alpha_s/\pi)G^2|\rangle}{\rm GeV^4}$	C_6/GeV^6
0 loops		0.198 ± 0.100	-0.0218 ± 0.0034
one loop	0.618 ± 0.059	0.056 ± 0.019	-0.0061 ± 0.0033
two loops	1.192 ± 0.136	0.061 ± 0.019	-0.0065 ± 0.0034
three loops	1.565 ± 0.193	0.062 ± 0.019	-0.0066 ± 0.0034

condensates in 0–3 loops approximation from analysis of the ρ-meson family are presented in Tables III and IV.

Since $\rho(770)$ has a small admixture of isospin $I = 0$, the decay $\rho \to \pi^+\pi^-\pi^0$ exists with the width

$$\Gamma_{\rho\to\pi^+\pi^-\pi^0} = \lambda^2\Gamma_{\omega\to\pi^+\pi^-\pi_0} = (7.23 \pm 1.20)\ \text{keV}. \quad (60)$$

The value

$$\Gamma_{\rho\to\pi^+\pi^-\pi^0}/\Gamma_{\rm tot} = (4.8 \pm 0.8) \times 10^{-5} \quad (61)$$

is smaller than the experimental restriction [30]

$$(\Gamma_{\rho\to\pi^+\pi^-\pi^0}/\Gamma_{\rm tot})_{\rm exp} < 1.2 \times 10^{-4}. \quad (62)$$

VII. CALCULATION OF THE ELECTRONIC WIDTH $\rho(1450)$

From Eq. (26) we have

$$\Gamma_1^{ee} = \frac{\alpha^2}{6\pi}\frac{[1 + r(s_1)]}{M_1}s_1^{(1)} \quad (63)$$

and using $s_1^{(1)}$ we get from (32)

$$\Gamma_1^{ee} = (2.57 \pm 0.31)\ \text{keV}, \qquad (\Lambda_3 = 1.565\ \text{GeV}) \quad (64a)$$

$$\Gamma_1^{ee} = (2.46 \pm 0.30)\ \text{keV}, \qquad (\Lambda_3 = 0.618\ \text{GeV}). \quad (64b)$$

The results presented in (54) and (55) are consistent with the value $\Gamma_1^{ee} = (2.5 \pm 0.9)$ keV obtained in [31].

VIII. CALCULATION OF THE ELECTRONIC WIDTH $\omega(1420)$

We have from Eq. (39)

$$\Gamma_1^{ee} = \frac{\alpha^2}{54\pi}\frac{[1 + r(s_1)]}{M_1}s_1^{(1)} \quad (65)$$

and using $s_1^{(1)}$ from (40) we obtain

$$\Gamma_1^{ee} = (0.27 \pm 0.03)\ \text{keV}, \qquad (\Lambda_3 = 1.565\ \text{GeV}) \quad (66a)$$

$$\Gamma_1^{ee} = (0.26 \pm 0.03)\ \text{keV}, \qquad (\Lambda_3 = 0.618\ \text{GeV}). \quad (66b)$$

Within the errors, the electronic width of the $\omega(1420)$ meson is one-ninth of the electronic width of the $\rho(1450)$ and is inconsistent with the value $\Gamma_1^{ee} = (0.15 \pm 0.4)$ keV [31].

IX. CONCLUSION

In summary, we present the results of the averaging over the ρ and ω families of the gluon and 4-quark condensates in the 0–3 loop approximation in Tables V and VI.

It is seen from Tables I, II, III, IV, V, and VI that the expansion of the gluon and 4-quark condensates in terms of α_s is very good. The good convergence in α_s is due to the improved perturbative theory [6,32,33] used in the paper.

The magnitude of the gluon condensate from the analysis of families of J/Ψ and Υ mesons was obtained in paper [3]:

$$0.04 \le \langle(\alpha_s/\pi)G^2\rangle \le 0.105\ \text{GeV}. \quad (67)$$

The first-order terms in α_s and the Coulomb terms of all orders in α_s/v_k are taken into account when obtaining (67). The result (67) is in slightly better agreement with $\Lambda_3^{\rm new}$ than with $\Lambda_3^{\rm conv}$.

The conventional magnitude of gluon condensate is the value obtained by Shifman, Vainstein, and Zakharov in their basic paper [11] from analysis of the J/Ψ family

$$\langle(\alpha_s/\pi)G^2\rangle = 0.012\ \text{GeV}^4. \quad (68)$$

TABLE VI. The averaged results of the calculation of the gluon and 4-quark condensates in 0–3 loops approximation for $\Lambda_3^{\rm new}$ from analysis of the ρ and ω families taking into account the $\rho - \omega$ interference.

	$\Lambda_3^{\rm new}$/GeV	$\frac{\langle 0|(\alpha_s/\pi)G^2|\rangle}{\rm GeV^4}$	C_6/GeV^6
0 loops		0.198 ± 0.100	-0.0218 ± 0.0034
one loop	0.370 ± 0.019	0.088 ± 0.018	-0.0096 ± 0.0030
two loops	0.539 ± 0.025	0.096 ± 0.018	-0.0105 ± 0.0029
three loops	1.618 ± 0.029	0.100 ± 0.018	-0.0108 ± 0.0029

But as was shown in Ref. [3] the model used by Shifman *et al.* in [11] to describe the experimental function $R_c(s)$ in the form of the sum of δ functions (related to the observed resonances from the J/ψ family) and a plateau contradicts the Wilson operator expansion in terms which are due to the gluon condensate. Later, there were many attempts to determine the gluon condensate by considering various processes within various approaches [32,34–52]. But in these works either R_c contradicts the Wilson operator expansion or Λ_3 is very small, $\Lambda_3 \sim 100$ MeV.

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